

1. An emission-control device is tested to see if it reduces CO<sub>2</sub> emissions from cars. The emissions from 6 randomly selected cars are measured with and without the device. The results are as follows.

Car	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
Emissions without device	151.4	164.3	168.5	148.2	139.4	151.2
Emissions with device	148.9	162.7	166.9	150.1	140.0	146.7

- (a) State an assumption that needs to be made in order to carry out a *t*-test in this case. (1)
- (b) State why a paired *t*-test is suitable for use with these data. (1)
- (c) Using a 5% level of significance, test whether or not there is evidence that the device reduces CO<sub>2</sub> emissions from cars. (8)
- (d) Explain, in context, what a type II error would be in this case. (2)

(Total 12 marks)

2. The weights of the contents of jars of jam are normally distributed with a stated mean of 100 g. A random sample of 7 jars was taken and the contents of each jar, *x* grams, was weighed. The results are summarised by the following statistics.

$$\sum x = 710.9, \quad \sum x^2 = 72\,219.45.$$

Test at the 5% level of significance whether or not there is evidence that the mean weight of the contents of the jars is greater than 100 g. State your hypotheses clearly.

(Total 8 marks)

3. An engineer decided to investigate whether or not the strength of rope was affected by water. A random sample of 9 pieces of rope was taken and each piece was cut in half. One half of each piece was soaked in water overnight, and then each piece of rope was tested to find its strength. The results, in coded units, are given in the table below

Rope no.	1	2	3	4	5	6	7	8	9
Dry rope	9.7	8.5	6.3	8.3	7.2	5.4	6.8	8.1	5.9
Wet rope	9.1	9.5	8.2	9.7	8.5	4.9	8.4	8.7	7.7

Assuming that the strength of rope follows a normal distribution, test whether or not there is any difference between the mean strengths of dry and wet rope. State your hypotheses clearly and use a 1% level of significance.

(Total 8 marks)

4. A certain vaccine is known to be only 70% effective against a particular virus; thus 30% of those vaccinated will actually catch the virus. In order to test whether or not a new and more expensive vaccine provides better protection against the same virus, a random sample of 30 people were chosen and given the new vaccine. If fewer than 6 people contracted the virus the new vaccine would be considered more effective than the current one.

- (a) Write down suitable hypotheses for this test. (1)
- (b) Find the probability of making a Type I error. (2)
- (c) Find the power of this test if the new vaccine is
- (i) 80% effective,
  - (ii) 90% effective. (3)

An independent research organisation decided to test the new vaccine on a random sample of 50 people to see if it could be considered more than 70% effective. They required the probability of a Type I error to be as close as possible to 0.05.

- (d) Find the critical region for this test. (2)
- (e) State the size of this critical region. (1)
- (f) Find the power of this test if the new vaccine is
- (i) 80% effective,
  - (ii) 90% effective. (2)
- (g) Give one advantage and one disadvantage of the second test. (2)
- (Total 13 marks)**

5. Gill, a member of the accounts department in a large company, is studying the expenses claims of company employees. She assumes that the claims, in £, follow a normal distribution with mean  $\mu$  and variance  $\sigma^2$ . As a first stage in her investigation she took the following random sample of 10 claims.

30.85, 99.75, 142.73, 223.16, 75.43, 28.57, 53.90, 81.43, 68.62, 43.45.

- (a) Find a 95% confidence interval for  $\mu$ . (6)

The chief accountant would like a 95% confidence interval where the difference between the upper confidence limit and the lower confidence limit is less than 20.

- (b) Show that  $\frac{\sigma^2}{n} < 26.03$  (to 2 decimal places), where  $n$  is the size of the sample required to achieve this. (3)

Gill decides to use her original sample of 10 to obtain a value for  $\sigma^2$  so that the chance of her value being an underestimate is 0.01.

- (c) Find such a value for  $\sigma^2$ . (3)
- (d) Use this value for  $\sigma^2$  to estimate the size of sample the chief accountant requires. (2)
- (Total 14 marks)**

6. An educational researcher is testing the effectiveness of a new method of teaching a topic in mathematics. A random sample of 10 children were taught by the new method and a second random sample of 9 children, of similar age and ability, were taught by the conventional method. At the end of the teaching, the same test was given to both groups of children.

The marks obtained by the two groups are summarised in the table below.

	New method	Conventional method
Mean ( $\bar{x}$ )	82.3	78.2
Standard deviation ( $s$ )	3.5	5.7
Number of students ( $n$ )	10	9

- (a) Stating your hypotheses clearly and using a 5% level of significance, investigate whether or not
- (i) the variance of the marks of children taught by the conventional method is greater than that of children taught by the new method, (4)
- (ii) the mean score of children taught by the conventional method is lower than the mean score of those taught by the new method. (6)

[In each case you should give full details of the calculation of the test statistics.]

- (b) State any assumptions you made in order to carry out these tests. (1)
- (c) Find a 95% confidence interval for the common variance of the marks of the two groups. (5)
- (Total 16 marks)**

7. A statistics student is trying to estimate the probability,  $p$ , of rolling a 6 with a particular die. The die is rolled 10 times and the random variable  $X_1$  represents the number of sixes obtained.

The random variable  $R_1 = \frac{X_1}{10}$  is proposed as an estimator of  $p$ .

- (a) Show that  $R_1$  is an unbiased estimator of  $p$ .

(1)

The student decided to roll the die again  $n$  times ( $n > 10$ ) and the random variable  $X_2$  represents the number of sixes in these  $n$  rolls. The random variable  $R_2 = \frac{X_2}{n}$  and the random variable  $Y = \frac{1}{2}(R_1 + R_2)$ .

- (b) Show that both  $R_2$  and  $Y$  are unbiased estimators of  $p$ .

(2)

- (c) Find  $\text{Var}(R_2)$  and  $\text{Var}(Y)$ .

(3)

- (d) State giving a reason which of the 3 estimators  $R_1$ ,  $R_2$  and  $Y$  are consistent estimators of  $p$ .

(2)

- (e) For the case  $n = 20$  state, giving a reason, which of the 3 estimators  $R_1$ ,  $R_2$  and  $Y$  you would recommend.

(4)

The student's teacher pointed out that a better estimator could be found based on the random variable  $X_1 + X_2$ .

- (f) Find a suitable estimator and explain why it is better than  $R_1$ ,  $R_2$  and  $Y$ .

(6)

**(Total 18 marks)**

1. (a) The differences are normally distributed B1 1
- (b) The data is collected in pairs or small sample size and variance unknown or samples not independent B1 1

**Note**

Allow because the same car has been used

- (c)  $d: 2.5, 1.6, 1.6, -1.9, -0.6, 4.5$  at least 2 correct M1
- $(\Sigma d = 7.7, \Sigma d^2 = 35.59) \bar{d} = \pm 1.2833, sd = 2.2675. (Var = 5.141)$  A1 A1
- $H_0: \mu_d = 0, H_1: \mu_d > 0 (H_1: \mu_d < 0 \text{ if } d - 2.5, -1.6, -1.6 \text{ etc})$  both depend on their d's B1
- $t = \frac{\pm 1.2833\sqrt{6}}{2.2675} = \pm 1.386\dots\dots$  formula and substitution, 1.38 – 1.39 M1 A1
- Critical value  $t_5(5\%) = 2.015$  (1 tail) B1
- Not significant. Insufficient evidence to support that the device reduces CO<sub>2</sub> emissions. A1 ft 8

**Note**

awrt  $\pm 1.28, 2.27$

- (d) The idea that the device reduces CO<sub>2</sub> emissions has been rejected when in fact it does reduce emissions. B1 B1 2
- OR
- Concluding that the device does not reduce emissions when in fact it does
- (if not in context can get B1 only)

[12]

2.  $H_0: \mu = 100, H_1: \mu > 100$  B1

$\bar{x} = \frac{710.9}{7} = 101.5571\dots; s^2 = \frac{72219.45 - \frac{(710.9)^2}{7}}{6}$  B1, M1

$s^2 = 3.746\dots \text{ or } s = 1.9355$  A1

test statistic  $t = \frac{101.557 - 100}{\frac{1.936}{\sqrt{7}}} = \text{awrt } 2.13$  M1 A1

$t_6$  5% 1-tail critical value = 1.943 B1 ft

Significant result. Reject  $H_0$ , there is evidence that the mean weight is more than 100g. A1 8

[8]

3.  $D = \text{dry—wet}$   $H_0: \mu_D = 0, H_1: \mu_D \neq 0$  B1

$d: 0.6, -1, -1.9, -1.4, -1.3, 0.5, -1.6, -0.6, -1.8$  M1

$\bar{d}: -\frac{8.5}{9} = -0.94, s_d^2 = \frac{15.03 - 9 \times (\bar{d})^2}{8} = 0.87527\dots$  A1, A1

$t = \frac{-0.94}{\frac{s_d}{\sqrt{9}}} = \text{awrt } -3.03$  M1, A1

$t_8$  2-tail 1% critical value = 3.355 B1

Not significant – insufficient evidence of a difference between mean strength A1 ft 8

[8]

4. (a)  $H_0: p = 0.3$  (or 0.7)  $H_1: p < 0.3$  (or  $> 0.7$ ) B1 1

(b) Let  $X = \text{number who contract virus}$ . Under  $H_0$   $X \sim B(30, 0.3)$

$P(\text{Type I error}) = P(X < 6 | p = 0.30) = P(X \leq 5) = 0.0766$  M1 A1 2

(c) (i) Power =  $P(Y \leq 5 | Y \sim B(30, 0.2)) = 0.4275$  M1 A1

(ii) Power =  $P(Y \leq 5 | Y \sim B(30, 0.1)) = 0.9268$  A1 3

(d) Let  $C = \text{number who contract virus}$ . Under  $H_0$   $C \sim B(50, 0.3)$

We require  $c$  such that  $P(C \leq c) \approx 0.05$  M1

$P(C \leq 10) = 0.0789, P(C \leq 9) = 0.0402 \therefore \text{critical region is } C \leq 9$  A1 2

(e) Size = 0.0402 B1 1

(f) (i) Power =  $P(D \leq 9 | D \sim B(50, 0.2)) = 0.4437$  B1

(ii) Power =  $P(D \leq 9 | D \sim B(50, 0.1)) = 0.9755$  B1 2

(g) Advantage: second test is more powerful

Disadvantage: second test involves greater sample size,  $\therefore$  more expensive or takes longer B1 B1 2

[13]

5. (a)  $\bar{x} = \frac{847.89}{10} = 84.79$ ;  $s_x^2 = \frac{103712.6151 - (847.89)^2 / 10}{9}$  B1  
 $s_x^2 = 3535.6522\dots$  B1  
 or  $s_x = 59.461\dots$   
 2.262 B1  
 95% confidence interval for  $\mu = 84.79 \pm 2.262 \times \frac{59.461}{\sqrt{10}} = (42.25, 127.33)$   
*accept (42.3, 127.3)* M1, A1, A16
- (b) 95% confidence interval  $\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$   
 so chief accountant requires  $1.96 \frac{\sigma}{\sqrt{n}} < 10$  M1 A1  
 i.e.  $\frac{\sigma^2}{n} < \left(\frac{10}{1.96}\right)^2 = 26.0308\dots = 26.03$  (2 d.p.) A1 cso 3
- (c) Require the upper confidence limit of 98% confidence interval for  $\sigma^2$   
 $\chi_9^2 = 2.088$ ; i.e.  $\frac{9s^2}{\sigma^2} > 2.088, \Rightarrow \sigma^2 < 15239.88\dots$   
*awrt 15240* B1; M1, A13
- (d) Substitute into part (b),  $n > \frac{15240}{26.03} \Rightarrow n = 586$  M1, A1 2

[14]

6. (a) (i)  $H_0: \sigma_C^2 = \sigma_N^2, H_1: \sigma_C^2 > \sigma_N^2$  B1  
 $\frac{s_C^2}{s_N^2} = \frac{5.7^2}{3.5^2} = 2.652\dots$ ;  $F_{8,9}$  (5%) critical value = 3.23M1; B1  
 Not significant so do not reject  $H_0$  – insufficient evidence A1 ft 4  
 that variance using conventional method is greater

(ii)  $H_0: \mu_N = \mu_C$   $H_1: \mu_N > \mu_C$  B1

$$s^2 = \frac{8 \times 5.7^2 + 9 \times 3.5^2}{17} = \frac{370.17}{17} = 21.774\dots$$

M1

Test statistic  $t = \frac{82.3 - 78.2}{\sqrt{21.774\dots(\frac{1}{9} + \frac{1}{10})}} = 1.9122\dots$

*awrt 1.91* M1 A1

$t_{17}$  (5%) 1-tail critical value = 1.740 B1

Significant – reject  $H_0$ . There is evidence that new style leads to an increase in mean A1 ft 6

(b) Assumed population of marks obtained were normally distributed B1 1

(c) Unbiased estimate of common variance is  $s^2$  in (ii)

$$7.564 < \frac{17s^2}{\sigma^2} < 30.191$$

B1 M1 B1

$$\sigma^2 > \frac{17 \times 21.774\dots}{30.191} = 12.3 \text{ (1 d.p.)}$$

A1

$$\sigma^2 < \frac{17 \times 21.774\dots}{7.564} = 48.9 \text{ (1 d.p.)}$$

A1 5

[16]

7. (a)  $X_1 \sim B(10, p) \therefore E(X_1) = 10p \Rightarrow E(R_1) = E\left(\frac{X_1}{10}\right) = \frac{10p}{10} = p$  B1 1

(b)  $X_2 \sim B(n, p) \therefore E(X_2) = np \Rightarrow E(R_2) = E\left(\frac{X_2}{n}\right) = \frac{np}{n} = p$  B1

$$E(Y) = E\left(\frac{1}{2} [R_1 + R_2]\right) = \frac{1}{2} [E(R_1) + E(R_2)] = \frac{1}{2} [p + p] = p$$

B1 2

(c)  $\text{Var}(R_2) = \frac{1}{n^2} \text{Var}(X_2) = \frac{np(1-p)}{n^2} = \frac{p(1-p)}{n}$  B1

$$\text{Var}(R_1) = \frac{p(1-p)}{10} \therefore \text{Var}(Y) = \frac{1}{4} [\text{Var}(R_1) + \text{Var}(R_2)],$$

M1

$$= \frac{1}{4} \left[ \frac{p(1-p)}{10} + \frac{p(1-p)}{n} \right]$$

A1 3

(d) Since  $\text{Var}(R_2) = \frac{p(1-p)}{n} \rightarrow 0$  as  $n \rightarrow \infty$ ,  $\therefore R_2$  is consistent M1, A1 2

(e)  $\text{Var}(R_1) = \frac{p(1-p)}{10} > \frac{p(1-p)}{20} = \text{Var}(R_2)$  M1

$$\text{Var}(Y) = \frac{p(1-p)}{4} \left[ \frac{1}{10} + \frac{1}{20} \right] = \frac{p(1-p)}{80} \times 3 < \text{Var}(R_2)$$

Since all 3 are unbiased, we select the one with minimum variance, i.e.  $Y$  A1 2

(f)  $X_1 + X_2 \sim B(n + 10, p)$  so consider  $\frac{X_1 + X_2}{n + 10}$  B1

$$E\left(\frac{X_1 + X_2}{n + 10}\right) = \frac{(n + 10)p}{(n + 10)} = p$$

*(show unbiased)* M1

$$\text{Var}\left(\frac{X_1 + X_2}{n + 10}\right) = \frac{p(1-p)}{n + 10}$$

*(find variance)* M1

$$\frac{p(1-p)}{n + 10} < \frac{p(1-p)}{10} \quad \therefore \text{always better than } R_1$$

And both A1

$$\frac{p(1-p)}{n + 10} < \frac{p(1-p)}{n} \quad \therefore \text{always better than } R_2$$

$$\frac{p(1-p)}{n + 10} < \frac{p(1-p)}{4} \left[ \frac{n + 10}{10n} \right]$$

$$\Leftrightarrow 40n < 100 + 20n + n^2$$

$$\Leftrightarrow 0 < 10^2 - 20n + n^2$$

$$\Leftrightarrow 0 < (10 - n)^2$$

*Show better than Y*

*Use of n = 20 acceptable* M1

$\therefore \frac{X_1 + X_2}{n + 10}$  is unbiased and always has smaller variance A1 cso 6

[16]

1. In part (a) few candidates realise that it is the “differences” which needs to be normally distributed and not the distributions themselves. In part (b) many candidates were able to write in some form that the data was collected in pairs. In part (c) the most common error was an incorrect standard deviation. The majority of candidates were able to apply the method correctly.

In part (d) it was pleasing to see that many candidates were able to write in context what the type two error would be although a few could only regurgitate the definition from a book showing that they had no real understanding of what it is.

2. No Report available for this question.

3. No Report available for this question.

4. No Report available for this question.

5. No Report available for this question.

6. No Report available for this question.

7. No Report available for this question.